MYSTERY CODE ANSWERS

11	10000	21	1010
12	12	22	10001
13	100000	23	100000000
14	1001	24	13
15	110	25	200
16	4	26	100001
17	1000000	27	30
18	21	28	1002
19	10000000	29	1000000000
20	102	30	111

Most people solve the code this way:

Prime numbers look like powers of ten. Beginning with 1 as the code for 2, you attach another zero to the "1" for each successive prime number.

2	1
3	10
5	100
7	1000, etc.

To find the codes for other numbers, factor them in any way you like and then add code numbers for these factors. For example,

$18 = 6 \cdot 3$	$11 + 10 = 21$
$18 = 2 \cdot 9$	$1 + 20 = 21$
$18 = 2 \cdot 3 \cdot 3$	$1 + 10 + 10 = 21$

No matter how you factor the number 18, the sum of the codes of the factors will be 21.

I originally designed this as a "prime factorization" code:*

Suppose the prime factorization of a number is written in exponential form, with the bases decreasing from left to right. The exponents in the prime factorization will give the code number. (All prime factors less than or equal to the greatest one must be represented, using "0" and "1" exponents when necessary.)

For example, the prime factorization of 1960 is $7^2 \cdot 5^1 \cdot 3^0 \cdot 2^3$. Reading the exponents from left to right, we see that its code number is 2103.

The number 0 has no code because it is not a natural (counting) number. It has no prime factorization.

*I was interested to discover that science fiction author Joan Slonczewski had earlier described a variant of this "code" in her novel "Children Star".

Notes:

- Many people find partial solutions by noticing "patterns within the main pattern". For example, they might notice that the powers of 2 (1, 2, 4, 8, 16, etc.) have the code numbers 0, 1, 2, 3, 4. . . – or that that powers of 3 (1, 3, 9, 27. . .) have the code numbers 0, 10, 20, 30. . .

- If you make *block diagrams* (see other activities in this site) for the original numbers, then the code simply counts the number of blocks of the different colors. Using the example above, the number 1960 has the code 2103 because its block diagram contains 2 yellow (7) blocks, 1 orange (5) block, 0 red (3) blocks and 3 white (2) blocks.

- The code runs into trouble for the first time when the original number is 1024. This is because $1024 = 2^{10}$. Can you see what the problem is? It's related to the fact that 1024 is the smallest natural number that has a two-digit exponent in its prime factorization.

- The reason that code for a composite number can be found by adding codes of its factors is related to the property of exponents, $a^m \cdot a^n = a^{m+n}$. When you multiply two numbers, the *counts* of each prime factor (the exponents of those factors) add.